SEISMIC INVESTIGATION OF THE SOLAR STRUCTURE USING GONG FREQUENCIES

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We investigate the effect of equation of state and convection formalism on the sound speed in solar interior by using the asymptotic inversion technique of Christensen-Dalsgaard, Gough & Thompson (1989). In this technique the sound speed difference between two solar models is estimated from the known frequency differences using,

$$S(w)\frac{\delta\omega}{\omega} = S(w)\frac{\omega_0-\omega}{\omega_0} = H_1(w) + H_2(\omega), \qquad (1)$$

where

$$S(w) = \int_{r_t}^{R_\odot}\left(1-\frac{c_0^2}{w^2r^2}\right)\frac{dr}{c_0}, \qquad (2)$$

ω_0 is the frequency of p-mode in the reference model, ω is the observed frequency for the same mode and $w = \omega/(\ell+1/2)$. The function $H_1(w)$ which is related to the sound speed difference between the model and the Sun, provides another measure of the sound speed difference. Similarly, $H_2(\omega)$ which reflects the differences in the surface layers is better suited to test different formulations of solar convection theory. The sound speed difference between the Sun and model can be found using the relation

$$\frac{\delta c}{c} = \frac{c_0-c}{c} = -\frac{2r}{\pi}\frac{da}{dr}\int_a^{a_s}\frac{\frac{dH_1}{dw}w\,dw}{(a^2-w^2)^{1/2}}, \qquad (3)$$

where $a = c/r$ and $a_s = a(R_\odot)$. As an independent diagnostic for differences in the surface layers we also consider the scaled frequency difference between the models and the Sun defined by $\Delta_{n,l} = Q_{n,l}\delta\omega$, where $Q_{n,l} = E_{n,l}/\bar{E}_0(w_{n,l})$, $E_{n,l}$ is the mode energy and $\bar{E}_0(\omega_{n,l})$ is the value of $E_{n,l}$ for $\ell = 0$ interpolated to the frequency $\omega_{n,l}$.

Table 1: Properties of solar models

Model	Conv	EOS	X	$r_d/R_\odot$	ρ_c
M1	CM	OPAL	0.7275	0.71643	153.0
M2	MLT	OPAL	0.7275	0.71659	153.1
M3	CM	MHD	0.7267	0.71761	153.2
M4	MLT	MHD	0.7267	0.71776	153.2

For this investigation we construct different solar models having identical physics except for the equation of state and treatment of convection. All the models use OPAL opacities (Rogers & Iglesias 1992) and incorporate diffusion of helium and heavy elements using the hydrogen abundance profile from Bahcall and Pinsonneault (1992) and heavy element abundance profile from Proffitt (1994). The convective flux has been calculated using either the usual Mixing Length Theory (MLT) or Canuto & Mazzitelli (1991) (CM) formulation. The properties of these models with heavy element abundance Z=0.019, are summarised in Table 1. These models use either MHD (Mihalas et al. 1988) or OPAL (Rogers et al. 1996) equations of state.
The error estimate due to the uncertainties in the observed frequencies is obtained by simulating 30 sets of frequencies where random errors with standard deviation quoted by the observers are added to the model frequencies. The variance in the computed values of relative sound speed difference gives the error estimate. The frequencies used are from the GONG network (Hill et al. 1996) spanning month 7, month 8 and those derived from the averaged spectra of months 4 to 7.

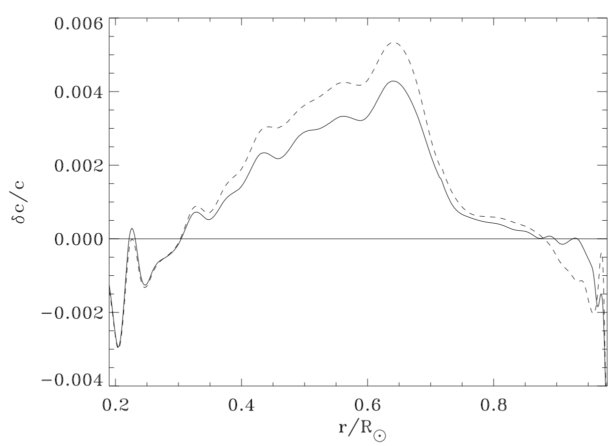

Figure 1: Relative sound speed difference between the Sun and models constructed using OPAL (solid line) and MHD (dashed line) equations of state. The hump near $r/R_\odot = 0.95$ is visible for the MHD equation of state

From the tests performed on inversion results it is clear that the differential asymptotic inversion technique is capable of providing reasonable results in the range $0.2R_\odot < r < 0.95R_\odot$. Figure 1 shows the rel-

ative difference in sound speed between the Sun and models M1 and M3. It is evident from the figure that the sound speed in model M1 using OPAL equation of state is closer to that in the Sun as compared to model M3 which uses MHD equation of state. Most of this difference in the radiative interior has come from the difference in depth of the convection zone between the two models. We can also notice a hump around $r = 0.95R_\odot$ inside the convection zone in MHD model. Thus it appears that the OPAL equation of state better describes the solar material which also confirms earlier results (Basu & Antia 1995a; Däppen 1996).
The two formalisms for stellar convection were tested by performing inversions using models M1 and M2. It appears that the model M1, which uses the CM formulation for calculating the convective flux is closer to the Sun as compared to the MLT model. However, a definite conclusion can not be made on the basis of inversion results as most of the difference occurs in the surface layers where the inversions are not particularly reliable.

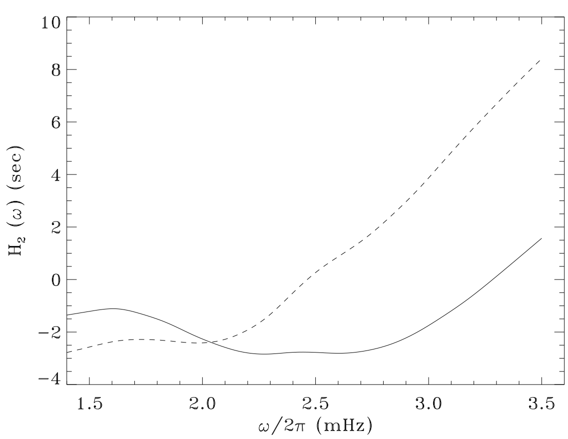

Figure 2: The function $H_2(\omega)$ between the Sun and models constructed with different treatment of convection. The sold line represents CM model and the dashed line MLT model.

Figure 2 shows $H_2(\omega)$ as a function of $\omega/2\pi$ for models M1 and M2. We find that model M1 using CM formalism has a significantly smaller variation than model M2 using MLT formulation. In particular, in CM model $H_2(\omega)$ is practically flat for $\omega < 3$ mHz probably indicating that CM models are closer to the Sun than models constructed using MLT. The scaled frequency difference between the models M1 and M2 and the Sun is plotted in Figure 3. It is evident that the scaled frequency difference between the CM model and the Sun is smaller than that between the MLT model and the Sun. Therefore, we conclude that the models using CM formulation for convection are closer to the Sun and further confirm earlier results (Paterno *et al.* 1993, Basu & Antia 1994, 1995b).

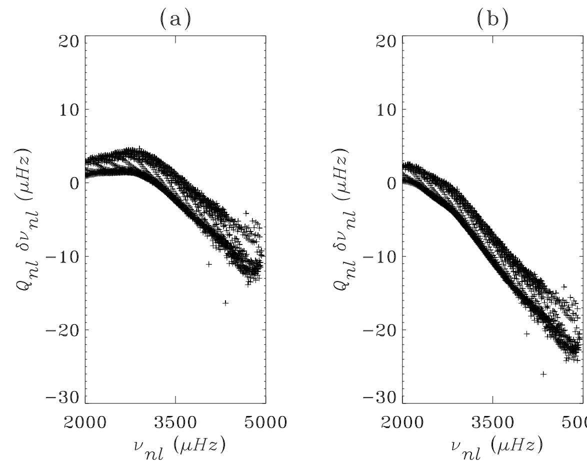

Figure 3: The scaled frequency difference between the Sun and (a) model M1, and (b) model M2.

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